DEVELOPING MATHEMATICAL RESILIENCE IN SCHOOL-STUDENTS WHO HAVE EXPERIENCED REPEATED FAILURE

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Abstract

Mathematics qualifications in the UK and many other countries represent valued cultural capital. In the UK, the typical qualification sought for employment in teaching, nursing, policing and many other professions is a GCSE award (minimum grade C) in mathematics. Although GCSE is typically taken at age 16, there is no logical or statutory reason why the award cannot be gained earlier or later. The UK government has recently determined that any student aged 16 to 19 who has not achieved at least grade C in GCSE mathematics should be enrolled on an approved mathematics course as part of their programme. Many students repeatedly fail to pass the examination; often such students re-sit the examination several times. We hypothesised that students, faced with a re-sit in mathematics, who complete a course to develop their mathematical resilience at the beginning, would be more likely to achieve the desired result.

The construct 'mathematical resilience' has been developed by Johnston-Wilder and Lee [1] to describe a positive stance towards learning mathematics. Mathematical resilience can be engineered within both formal and informal learning environments by a strategic and explicit focus on the culture of learning mathematics. Previous papers (for example, [2]) have described engineering the growth of mathematical resilience through training adult coaches for mathematical resilience to work alongside learners outside the school environment. This paper discusses a course in mathematical resilience; the course was versioned for school students in year 12, students who had repeatedly failed to achieve the required grade in GCSE mathematics, and who were now preparing to retake the examination yet again. This short course, which ran from September to November 2014, was focused on helping students to overcome affective barriers and develop more resilient strategies for working with mathematical ideas, rather than on memorising mathematics content. The 17 students had been given very strong direction by the school to attend this course; they were told that if they attended and subsequently failed GCSE mathematics again, they would have shown they were making the effort and future opportunities would be approved for them to re-sit, however if they did not attend and failed again, they would be asked to leave the school.

The course aimed to develop students' mathematical resilience, so that they could more effectively support one another when facing difficulties in mathematics. This work developed a culture of 'can do' mathematics to counter the prevalent culture of mathematics helplessness, failure and mathematics anxiety. Participants learned to consider and manage their own reactions to mathematical ideas, to explore choices and to reflect on how to support themselves and each other to overcome their barriers to learning mathematics. The data confirm that, once an individual has begun to develop their own mathematical resilience and has worked through their own anxieties and negative stance towards mathematics in a safe and collaborative environment, they can then successfully coach themselves and others to develop mathematical resilience. Outcomes for these learners will be discussed.

Keywords: mathematics anxiety, learned helplessness, mathematical resilience, peer coaching

1 THE LONG TAIL OF UNDER-ACHIEVEMENT

The latest Pisa study [3] showed England continued to have a long tail of underachieving students in mathematics. The gap between high and low achievers appeared to have widened. In the GCSE examinations that are intended to be taken from age 16, Grades G to D indicate success at National Level 1; grades C–A* indicate success at National Level 2. There was a large pool of people (estimated 23% of a cohort) who did not have Mathematics Level 2 and who are capable of attaining it. Most students who achieved a D grade in mathematics GCSE at secondary school did not enter the exam in post-16 education [4].

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In response to such concerns, the government recently determined that any student aged 16 to 19 who has not achieved at least grade C in GCSE mathematics should be enrolled on an approved mathematics course as part of their post-16 study programme, in order for that study to be funded. Students who had attained a grade D should re-sit the GCSE as a one-year programme. Sadly, the government did not appear to recognise the scale of the affective barriers involved. Pressure was applied to schools in the form of published performance tables, comparing each school with each other, and in the form of floor targets below which a school was at risk of being closed. This resulted in schools multiple-entering students allowing for more chances to attain the required grade. Further pressure on schools, and therefore on students, was brought about by a shortage of teachers of mathematics. In England, this resulted in mathematics deprivation being spread unequally across England. Thus a situation was engendered where many students repeatedly sat the examination and failed to gain a grade C.

The 'system' tends to blame the individual students for this, the prevalent discourse being that the students are not engaging and not making the effort to achieve, or they do not have 'ability'; however there is evidence to suggest students are avoiding engagement with mathematics in order to protect themselves from the harm that can be caused by repeated failure without personal agency or control [5]. Mathematics anxiety has been shown to be a significant contributing factor in both underachievement and reluctance to continue study of mathematics [4]. In some extreme cases, parents have insisted on students being withdrawn from post-16 mathematics courses because of the distress being caused at home.

2 WHAT HAPPENS WHEN A LEARNER ENCOUNTERS REPEATED FAILURE?

Psychological literature (for example, [6]) indicates that when a learner encounters repeated failure, there are two alternative consequences, namely increased determination, leading to improvement in performance, or emotional harm and helplessness, anxiety, inactivity, and fear, leading to reduced performance.

If the learner deems the process of getting better at mathematics to be within their personal control, and believes they are capable of changing the outcome, this can be intrinsically motivating and rewarding, then repeated failure in this circumstance can result in more determination for example when a learner is engaged in a computer game in which progress is based on repeated exposure to a scenario until certain skills are gained; this is what happens when the learner holds a 'growth mindset' [7]. If the learner deems the process of getting better to be outside their control, then the result can be helplessness and hopelessness, in particular, this is what happens when the learner holds a 'fixed mindset' [7] and believes that however much effort they put in, they do not have the ability to gain GCSE grade C [6].

"What has emerged from the collective lines of research is the understanding that the tendency of events to subsequently trigger some analogue of anxiety or negative affect is dependent to some degree on the amount of control the organism experiences over those events." [8] Thus, prior mathematics experience will affect responses to any new experience of mathematics. The 'primitive' brain stem is involved in physiological and emotional responses to stress, thus a fight, flight or freeze response is evoked in extremis. With increasing levels of emotional and physiological stress, prefrontal functioning decreases, cognitive control reduces and, consequently, there is increased risk of maladaptive behaviours, such as avoidance, helplessness and anxiety [5].

Identifying reliable and tractable methods for addressing mathematics anxiety seems crucial to increasing mathematics competencies [5]. Our previous research has shown that by creating opportunities to support the development of mathematical resilience, some individuals have been able to achieve significant improvements in their mathematical performance; this is provided that, where it has taken hold, mathematics anxiety is addressed explicitly. Where harm is more established, individuals can take longer to develop mathematical resilience.

2.1 A shocking example

Our shock at the situation in the intervention school was that many of the students had been entered repeatedly for an examination in which they experienced failure. There was significant evidence of lack of progress, or for some deterioration of grades between successive sittings of the examination. This was understandably having a harmful impact on the young people involved.

Through the lens of a growth mindset, this situation warrants investigation. However in our experience, this rarely happens, instead justifications are often given by statements such as “these students don’t have the ability” to do better, which seemed to be the case in this school, a clear indication of a fixed mindset approach.

It seems that most of these students were let down by the uneven distribution of mathematics teaching across the country. What was worse, the indicators of how they felt about themselves as a result of the environment they experienced were distressing. Students described themselves as: *feeling like rubbish (poo); feeling stupid; looking like a delinquent; letting my family down; stressed; in a vicious cycle of failure*. One student, when asked at the beginning of the course to describe a positive incident in a mathematics lesson responded: “*I don’t really think I’ve ever had a positive moment with maths. Not in a lesson or anything*.”

2.1.1 Evidence from before course started. Student comments.

Before the course started, in addition to looking at historic grades, we asked the students to prepare a short piece of writing “Maths and me so far”. The data from these short pieces was analysed and is reported here; it makes quite distressing reading.

Some students reported a lack of confidence: “*I have never been confident at maths”; “I haven’t been very good at maths, even when I was in primary school. My confidence isn’t very strong either.”* Thus for some, the lack of confidence goes back for over seven years. Other students reported not liking mathematics because: “*it is frustrating; I don’t understand; I feel stupid.”*

Others adopted a self-protection approach by not expending effort: “*I’m more of a creative person”; “it annoys me”; “I don’t get it”; “I give up too easily”; “I don’t really need it if I‘m honest”. Others implicitly criticised the teaching: “Why I can’t do maths: I just don’t understand maths; I might as well give up; pointless being in a lesson with everyone else, because I can’t keep up; can’t revise on my own (without a teacher) because I need to be shown how to actually do it*.” One explicitly mentioned boredom: “*The reason why I hate maths is my brain does not switch on when learning it. I don’t like it. It’s boring. I get distracted easily. I give up easily*.” During the course, school mathematics lessons were described as “*boring*”; “tiring” and “draining”; “*teachers don’t listen*”. Such comments evoke the findings of Nardi and Steward [9] that students who rejected school mathematics and left when it became optional, experienced school mathematics as T.I.R.E.D.: tedious, isolated, rote, elitist, de-personalised. One student was clear that they had experienced a largely structural problem rather than a personal one: “*I was good at maths in primary school, despite being in the bottom group; year 7 my teacher left and I ended up having [substitute teachers] for the rest of the year; year 8 I was alright, I was too distracted; year 9 teacher left and I only had [substitute teachers]; year 10 teacher left; year 11 I was too stressed to care, I got a D and a U*.” Several more went beyond TIRED and described further stress and hate: *“I give up easily if I get something wrong”; “I hope to pass maths for it stress me out”; “I hate maths”; “I can’t do it”; “I find it confusing”; “even though I answered every question I still failed”; “I give up easily with it".*

Some went further than stress and described fear and panic: “*My overall attitude and mind state before entering a mathematics lesson, pressure, fear of maths and embarrassment”;* One wrote: *“Even before I would enter a maths lesson or even start to answer a maths question ... I would panic*”. Some students wrote vividly about additional anxiety in mathematics exams: *“Leading up to my exam and in my exam I became stressed and panicked. This then caused me to forget everything, well the harder stuff.” “I have never been good at maths; I learn things then forget them; in exams, I panic and forget a lot of the stuff I’ve learnt.”* Of even greater concern was that these feelings also affected the students at home: “*I get really upset at home, and with my results and everything, because that’s the only lesson that I don’t do well in. I don’t do bad in anything else really, it’s kind of always been maths, so they know I struggle with it, and it upsets me.”*

Despite these past experiences, the relationship and the contract with the year 12 staff were such that all but one student who had not previously gained a grade C signed up for a course that involved 8 half days and staying after school for an hour a week. This is a big commitment for young people so disaffected in mathematics; they were clearly capable of being motivated.

2.2 Structural violence

We would suggest that these young people have been mathematically harmed and have significantly under-achieved. Previously, we have used the term ‘mathematical cognitive abuse’ to describe such harm. The term ‘mathematical cognitive abuse’ lays individual teachers open to feeling blamed; this was not our intention. Here we use the term ‘structural violence’, which is more evocative of harmful structures in place that could be removed if there were sufficient will and understanding. The concept of structural violence was developed initially by Johan Galtung [10], a pioneer of peace studies, to describe causes of harm where individuals are not at fault, but where structural processes constrain personal agency. *“Structural violence is one way of describing social arrangements that put individuals and populations in harm’s way… The arrangements are structural because they are embedded in the political and economic organization of our social world; they are violent because they cause injury [including mental and emotional injury]… [11]*

Thus the term ‘structural violence’ acts as an umbrella to include institutional failings that have harmful consequences in people’s lives, and prevent individuals from making the progress of which they are capable. Since much of scientific progress involves understanding mathematics, and mathematics is also used as a gatekeeper to many roles, such as nursing and level 3 child-care qualifications, we argue that the term ‘structural violence’ applies to those who have been excluded from mathematics unnecessarily, to the point of distress. Structural violence leaves people powerless, excluded, with a lack of personal agency to address the situation.

3 WHAT IS TO BE DONE

Recent UK government policy has begun to recognise the importance of the affective domain in learning mathematics. For example, Education Minister Elizabeth Truss (March, 2014) acknowledged the need for ‘confidence’. In addition, the Welsh government has asked parents to be careful what they say about mathematics, to help foster a more positive attitude. Students in England have low levels of perseverance in mathematics [3]. According to Dweck [7], we need to develop in young people a growth mindset and a focus on effort rather than ability. According to Bandura [12], learners need personal agency, “the capacity to exercise control over one’s own thought processes, motivation, and action” (p. 175) and ‘resilient self-efficacy’ to manage the affective domain when students meet unknowns and failure.

Galtung [10] uses the notion of personal agency; he suggests that the impact of structural violence can be addressed by talking therapies in which the harmed find a listener, and then find a voice by which to begin to remove the power of the system which is continuing to harm their inner identities, by re-framing. Human agency can also be exercised through collective experiences and culture of a group. Lyons and Beilock [13] say that rather than more mathematics courses, *educational interventions emphasizing control of negative emotional responses to math stimuli will be most effective in revealing a population of mathematically competent individuals, who might otherwise go undiscovered.”*

We have designed such an intervention [2], and show here how it can help excluded young people to achieve in mathematics. We call the intervention ‘developing for mathematical resilience’ through peer and self-coaching.

3.1 Mathematical resilience – a pragmatic response

Drawing upon Dweck’s notion of growth mindset [7], we have found it helpful for learners and coaches to think of mathematical resilience as what is needed to stay, safely, for as long as possible in the ‘growth zone’. This zone is immediately beyond what a person is able to do reliably, without aid or support. The growth zone model was further expanded in a previous paper [2], and the diagram is reproduced here as Fig. 1. It is our experience that this idea of a growth zone needs explicit teaching, to help learners overcome their prior experiences of mathematical harm, to become aware of their emotions, attitudes and beliefs, and to learn actively to manage mathematics anxiety.

Figure 1: The growth zone model

In the growth zone, learners make mistakes, go down dead-ends, experience some failure, require support, get stuck and find activity challenging and tiring. They learn that being in the growth zone may trigger productive levels of adrenalin, accompanied by feelings of excitement or mild anxiety. Ideally, students will feel motivated and appropriately supported, and will have opportunity to enter their growth zone often. The required learning environment is one of trust, courage, articulation, collaboration, persistence and the personal agency to ensure well-being. Many learners avoid going into their growth zone in traditional mathematics lessons, as they have learned from experience, they quickly find themselves beyond it and in the danger zone of social and psychological harm.

Since so much of everyday life uses mathematics, learners need to engage with mathematics. In order to do this many students need to know that they can safeguard themselves by recognising when they are likely to cross over into the danger zone and how they can find a way back to the growth or comfort zones. We argue that, to achieve this, they need personal agency, and a supportive community of co-learners, ideally supported by a coach. [2]

We found that mathematical resilience can be developed in people who have experienced previous mathematical exclusion or stress, through strategic and explicit focus on the culture of learning mathematics within both formal and informal learning environments. People who were mathematics-avoidant can become curious and increasingly aware of their feelings; they develop an internal locus of control and a strong social learning network, they learn to seek and give help. We propose that valued mathematics experiences, together with a language for awareness and management of risk, can help learners develop risk taking and management processes in their learning of mathematics so that they spend increased time in their mathematical growth zone. Learners need to experience protective factors: connection, competence, opportunities for participation and contribution. However, to impact on mathematical resilience, this does not all need to happen within the mathematics lesson.

3.2 Coaching for mathematical resilience

Any person, ‘mathematical’ or not, can support others with mathematics by understanding how each of the zones described above is experienced, and by encouraging explicitly the development of mathematical resilience, without taking responsibility for the mathematics content.

Coaches for mathematical resilience can help peers (and themselves) to recognise what zone they are in — danger, mathematical growth or cruising in the comfort zone — and to encourage peers to value challenge, to manage the emotions involved, to take action to safeguard themselves, and to help them think about and use resilient learning ideas when facing difficulties. A coach will support, respect, listen, be compassionate, validate, model resiliency, and refrain from judging, enabling a coachee to feel safe in exploring, thinking through options [14] and taking managed risks in order to grow mathematical capability. Previous research [2] has shown that coaches will need to learn to: use a language that allows peers to recognise and articulate what degree of challenge they are facing; encourage increasing independence and agency; model a strategy or model being part of a community of practice; know how to access help. A coach’s responsibility is to nurture a culture of ‘can do’ mathematics which works to counter the prevalent culture of mathematics helplessness and mathematics anxiety when faced with mathematical ideas. The coaches are not required to know the mathematics but rather to develop the peer’s ability to recognize options and actions that might yield an understanding of the mathematical ideas involved and thus lead the coachee to an answer.

The Coaching for Mathematical Resilience course (level 1) is designed and delivered by The University of Warwick and The Progression Trust and accredited by ASDAN. [2] It is not a course in mathematics, although mathematics is encountered in the process of becoming resilient and learning to support self and others. We use the coaching model proposed by Egan [14] to encourage the development of effective personal agency. During the course, students learn to coach themselves and others by employing active listening to explore a situation, consider options and then set and review goals, acknowledging how they might address hindrances and the potential to sabotage progress. The course is jointly led by an experienced coach and an experienced mathematics teacher. It is important to note that the coach does not need to know any mathematics; the coach focuses on teaching and modelling techniques, skills and dynamics of coaching, on the ‘mathematical safe-guarding’ of participants and on being mathematically resilient.

Challenges from the ASDAN Short Mathematics course are used as the focus for the coaching activities. The ASDAN Mathematics Short Course provides a curriculum and resources fully commensurate with the principles of both the construct of mathematical resilience and the coaching approach, through accessible, challenge-based learning, formative assessment, and the ‘plan-do-review' learner reflection process. The ASDAN Mathematics Short Course provided a framework that allows teachers to make any mathematical concept accessible, using appropriate enactive tasks when an iconic or symbolic version [15] is not yet accessible to learners; thus there is no cognitive reason for anyone to be excluded from developing mathematical thinking.

This paper discusses the outcomes of a course presentation in 2014 that recruited 17 participants, all of whom had experienced repeated failure at GCSE mathematics. Course participation was encouraged by a contract presented by the school, namely that students who took part would be able to continue into year 13 even if they failed to gain a GCSE grade C in the summer examination at the end of year 12. We supported this contract because we felt it supported "effort" not "achievement", without resorting to coercion.

4 THE JOURNEY (WHAT HAPPENED)

We have found it is effective at the beginning of the course to establish shared ground rules and encourage students to think about what helps them to feel safe enough to learn well. In the light of past experiences, the ground rules developed by the participants were to: "*do whatever you need to do to keep yourself safe; not laugh at people who make mistakes; mistakes are good – take risks; ask questions; not be shy; not be hungry or thirsty; interact – work together; not procrastinate– get on with it and have fun; be generous; be supportive; give each other space."* In the process of articulating these ground rules, the participants had articulated the need for a space in which to make mistakes, ask questions and give and receive support. This was prior to us introducing the growth zone diagram.

They were able to describe how they had undermined themselves in the past by: *"Remembering failure – holding on to it – nurturing it; telling yourself you are rubbish; not going to sessions; not putting in 100%; not turning up to an exam; saying 'It's not my fault'; finding distractions*."

They hoped that they would: "pass" mathematics, i.e. gain at least a grade C; change how they felt about mathematics, *be more confident in maths; to feel good about it*." However there were also strong negative emotions that had developed in relation to mathematics with some hoping never, ever, to do mathematics again. The students said they feared: *failing maths; failing and having to do it over and over again; to fail maths even though I participated in this programme; feeling like an idiot; failing maths and this to be a waste of time.* These comments were indicative of a widespread fear, combined with concern that their hopes for the course might prove unattainable.

Early in the course, explicit emphasis was placed on developing a supportive learning community, intended to foster curiosity in place of fear, to encourage participants to take risks and to encourage shared responsibility for learning in place of embarrassment about making mistakes. We had envisioned that to build a community of practice would take time. As a precaution, we planned the course over eight half-day sessions. This is shorter than the course described previously [2], because the students were all from the same school; we reasoned that they would build a community more quickly than participants from different institutions.

When introduced to the growth zone model, the students were invited to use words to describe how the zones had felt based on their own past experience. They described being in the green zone as: *knowing it; feeling comfortable; familiar; easy; no effort; confident.* They described being in the amber zone as feeling: *alert; exciting; curiosity; 'in the zone'; slightly scary; on edge; energy.* They described being in the red zone as feeling: *too much; not safe; risky; dangerous; too far; panic* and one student made a particularly pertinent comment: *take control!*

When students were asked to reflect on their current habits when they experienced the danger zone, it was apparent that they adopted active or self-soothing behaviours that many teachers might find odd or disruptive. These included swearing, throwing something, eating, covering their face or go on their phone. They did so to take their minds off the fear and panic triggered by the mathematics. The students subsequently learned to use coloured cards to communicate which zone they were in. This enabled the coach and the peers to see when the student needed additional support to stay safe and to manage any negative emotions.

Many of the students found the sessions to be hard work. They expressed feelings of insecurity and weakness, but acknowledged the comfort that came from knowing that others in the group felt the same. As the course unfolded, they began to recognize how their attitude and their state of mind, including fear and embarrassment, were hindering their progress with mathematics. They began to set themselves positive goals about getting involved, persisting with the course and developing confidence. They reported that in their regular mathematics lessons they had *"answered more maths questions, and attempted more than I would [have done] before the program"* and in some cases already *"Actually getting questions right and understanding."* They began explicitly to express pride in their progress.

4.1 What the group learned.

By the end of the program the students reported what they had learned that was particularly helpful. They had learned that it is ok to make mistakes and that if they were persistent and committed, they made pleasing progress. Students also became more aware of the importance of working with their peers rather than depending solely on the teacher for support. Some referred to having overcome feelings of shame or embarrassment, but were strongly aware now of how "*we could all help each other*". One student recalled: *when we [were] doing all the algebra and all working together on the maths paper, we all had different knowledge, so that when we put it together it turned out to be better.*

The students had learned to be more aware of their own emotional responses to mathematics. One student said*: I will now pay more attention to my emotions and fear to maths, and take into consideration my issues physically towards maths and also my ways of dealing and coping with the potential developing issues, [including] ask for help, talk to a friend.* In this respect, many of them reported finding the language of the growth zone model was helpful.

Many students valued the accessibility of the mathematics due to inclusion of enactive processes. One student said her favourite session was when she learned *how to do my 9 times tables with my fingers.* Another remembered as very positive a session in which students were asked to line up to represent a box-plot. It was significant time for the students since they had previously believed the idea of a box-plot was beyond their comprehension. Following this enactive exercise they all felt able to understand the concept: *it showed people that didn't know what they were doing how to do it in a more visual way so they'd understand it. I was feeling positive. I was a little bit unsure at first because I wasn't quite sure. I'd never done one before. But by the end of the session I knew what I was doing.*

In summary, they learned that rather than mathematics being necessarily TIRED [9], mathematics can be experienced as accessible, linked to real life, inclusive of them, valued by them as worthwhile, and engaging (ALIVE)

4.2 The impact of the intervention short-term

We measured the students' mathematics anxiety and mathematical resilience before and after the course. Mathematics anxiety was measured using the Betz [16] Mathematics Anxiety Scale (MAS). Mathematical resilience was measured using MRS [17]. The prior responses added evidence of a significant level of shame, anxiety, and exclusion that these students had experienced around mathematics. Following the intervention, measured mathematical resilience went up for each participant and measured mathematics anxiety went down (see Fig. 2).

The chances of this happening randomly are less than 1/64 (0.0156); thus even with such a small sample there is evidence to accept the hypothesis that this did not happened by chance. Furthermore, there was one student who completed the mathematics anxiety scale but did not participate in the course; in contrast with the general trend towards reduced anxiety, their measured mathematics anxiety went up from 24 to 27.

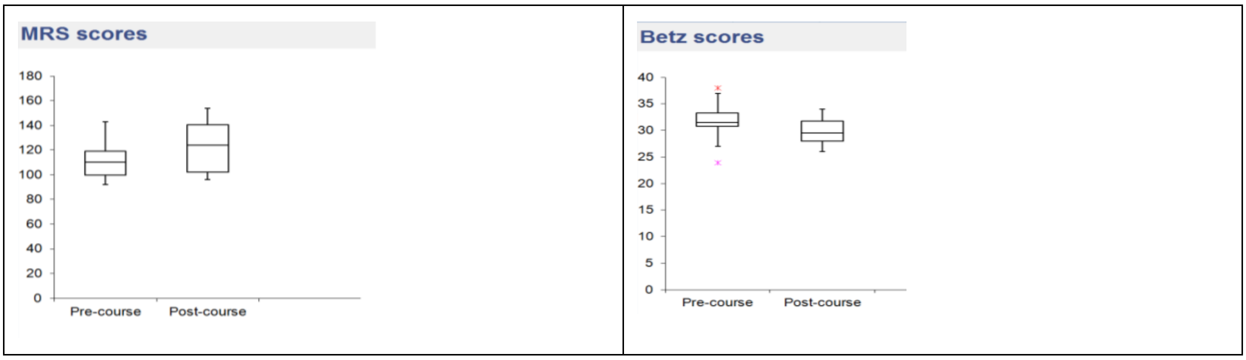

Figure 2: Measured scores pre- and post-intervention

4.3 Reflections on the transformation

In the interview at the end of the course, students were encouraged to reflect on what had changed for them. They described their pre-course self as not feeling they could be bothered, not wanting to try because *"I knew I probably couldn't do it". "I don't care. People think I'm dumb at maths."*

They described their post-course self as being more positive and confident, and becoming more involved in mathematics lessons. Some students actively sought more mathematics by requesting one-to-one tutoring or attending extra mathematics lessons. They were able to ask for help "*I'm not afraid to ask or say that I don't understand it"* without feeling embarrassed "*I didn't feel as embarrassed to admit that I was struggling*" and let others guide them to break down the problem "*"Well I can't do that" and he'll break it down for me and explain it better then I feel more…I tell him to break it down.* One student said: "*I went to see my maths teacher, and she's [going to] organise one-to-one maths tutoring for me. I should have done it a long time ago."*

Students reported that the CfMR approach helped them in other lessons and outside school too, when speaking with people and thinking about new ideas.

4.4 The impact of the intervention longer-term

There are two aspects to the longer-term impact of the intervention. Firstly, there was some impact upon the examination results in the following summer. For the students who returned after the summer we were told their results. Five participants recorded an increase on their recent grades, while another seven showed continued stagnation. For us this demonstrates a small success. This group of students had experienced long term mathematical neglect and had shown deeply entrenched negative emotions towards mathematics, and we expected that the intervention might take longer to make a significant impact upon exam results.

Secondly, participants who returned to the school the September 2015 showed evidence that their mindset had changed from helplessness to determination. The determination manifested in focused and proactive behaviour, asking about what mathematics tuition would be available and when the next re-sit exam would take place. The head of the sixth form reported being very impressed by the markedly more positive attitude shown.

5 DISCUSSION AND CONCLUSION

The course provided a safe and collaborative working environment in which the school students learned to manage their own reactions to mathematical ideas, to explore choices and to reflect on how to support themselves and others to find the resources to overcome affective barriers to learning mathematics. The transformations experienced by the participants have impacted more widely than study of mathematics, and this impact has been recognised both by the individuals and by the school.

5.1 Increased personal agency

The increase in personal agency begins with developing students' awareness of their emotional responses, and what they can do to manage any negative response. As we had previously found with older coaches [2], the notion of mathematical resilience as personal safe-guarding in the mathematical growth zone, and the language of red, amber and green zones to describe affective responses to situations, seem to have been very supportive in increasing personal agency, both in mathematics-specific situations and more generally. As peer coaches, the students learned to remain outwardly calm, protecting themselves and their peers by managing their own anxiety about mathematics. The coach does not "show how"; they encourage personal agency by asking question such as, *'what could you do?*' CfMR self- and peer-coaches learn to understand the emotions that learners can feel and how coaching can help.

5.2 Changes of habit and belief

By working as a community, the participants on the course provided one another with a safe and collaborative working environment, conducive to learning effectively. Participants reported that they: felt safe; worked together as a collective and in communal activity; had fun and enjoyed the learning; experienced success and achievement. There were significant changes in their habits and beliefs they reported an ability to listen to one another, to set their own goals and they showed aspects of a growth mindset in that they became willing to persevere, and to assertively ask for details of the re-sit class.

5.3 Re-framing experience of mathematics

The students developed understanding that the problems they were experiencing with mathematics did not arise because of their own inherent or unchangeable traits. They came to recognise that they are not '*stupid*' when it comes to mathematics, and that they are able to learn mathematics given their own effort, new skills in managing strong emotions rooted in safe-guarding, and the opportunity to recruit appropriate support. They found it helpful to recognise that they had been unnecessarily excluded and harmed, and that by taking control, working in groups, asking questions, and approaching coaches, they could use their innate and acquired powers to make progress in mathematical learning in the same way as they could in other aspects of their lives.

5.4 Recommendations

School mathematics in England currently fails too many children; for many, the problem "is only in mathematics." This is not a matter of 'ability'; every student in this research had shown their ability to learn in other subject areas, but of a lack of understanding of how to overcome strong negative emotions and learn effectively in mathematics. They had to develop a growth mindset, and their personal agency, so that they could set personal challenging, but attainable, goals, whilst recruiting appropriate support. It seems that by explicitly addressing mathematics anxiety and avoidance in similar ways to those reported here, recruitment into STEM-related careers has the potential to increase and progress will be improved in mathematical attainment for much of the population.

Those with the cultural capital of an existing mathematics qualification have coped with, and some prefer, mathematical learning environments to be TIRED [9]: tedious (predictable); isolated (individual); rote (formulaic); elitist (clever) and de-personalised (out of context). We ended our previous paper [2] by stating: "We suggest that anyone who has been historically excluded from mathematics, mathematically shamed, or who has developed mathematical anxiety or avoidance in an attempt to protect themselves from further mathematical harm, should feel entitled to such a life-changing experience."

They should also feel entitled to experience mathematics as ALIVE: accessible, linked to what is already known and understood, inclusive, valued (personally as well as culturally) and engaging. In this paper, we use the language of structural violence to describe the causes of mathematics anxiety and mathematics avoidance. We suggest that:

- As a community, we take mathematics anxiety and stress seriously and we explicitly create safe environments for learning mathematics.
- Two tutors, one a mathematics education specialist and the other a coaching specialist, collaborate in the delivery of the course to ensure that mathematical well-being can be adequately addressed and integrated into delivery.

In their own words, the students said: *I'm thankful. I do recommend people doing the course if they really do feel uncomfortable about maths and they struggle with it. Because it makes you feel more positive. I think it makes you feel a lot better about yourself.*

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